Phonons and internal friction in incommensurate composites

2002 J. Phys.: Condens. Matter 14 12411

(http://iopscience.iop.org/0953-8984/14/47/314)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **14** (2002) 12411–12422 PII: S0953-8984(02)37226-6

Phonons and internal friction in incommensurate composites

T Janssen

Institute for Theoretical Physics, University of Nijmegen, Nijmegen 6525ED, The Netherlands

Received 21 May 2002
Published 15 November 2002
Online at stacks.iop.org/JPhysCM/14/12411

Abstract

The non-linear dynamics involved in sliding modes in an incommensurate composite is studied in the double chain model. The coupling between the chains leads to phonons in which the two chains participate in various ways. For displacements beyond the harmonic approximation it is shown that, for a coupling small enough to leave the modulation functions continuous, there is a dynamic transition from a practically frictionless regime to a regime with strong dissipation. In the cross-over region resonances lead to an oscillatory motion of the subsystems with respect to each other. In the regime of discontinuous modulation functions there is strong dissipation and a transition from a pinned to an unpinned state. The unpinned motion is a non-linear wave in superspace.

1. Introduction

Aperiodic crystals may be described by embedding them into a higher-dimensional space as lattice periodic structures. In this higher-dimensional space it is possible to have more hydrodynamic degrees of freedom than just the dimensions of the physical space. In addition to the acoustic modes there may be displacements in the additional space, corresponding to so-called phason degrees of freedom. In contrast to the acoustic modes these extra modes, generally, have non-negligible dissipative character. In general, these modes are overdamped. For incommensurate composites, a special class of these aperiodic crystals, these modes are called sliding modes and correspond to mutual displacements of the constitutive subsystems. The damping of these sliding modes then can be considered as an internal friction.

We study this friction on a simple model, the double chain model (DCM) [1, 2]. It consists of two parallel one-dimensional chains with anharmonic intra- and inter-chain coupling. If the two chains are incommensurate, the coupling leads to incommensurate modulations of the chains in the ground state. In the dynamics the coupling leads to interaction of the phonons in the chains, changing the vibrational spectrum.

Beyond the harmonic approximation the dynamics is described by coupled phonons, but also by non-linear excitations of the chains.

The model may serve as well for the motion of two crystal structures with respect to each other, i.e. to the friction between two mutually incommensurate contact surfaces. The model used is a Hamiltonian system. Therefore, the energy is conserved. Dissipation here means that energy in one mode, in particular the centre of mass motion, is distributed over other modes where it forms the heat bath.

Friction between mutually incommensurate crystal surfaces has been studied mainly on the Frenkel–Kontorova model. This model describes the motion of a crystalline chain on a rigid periodic substrate. It was claimed that there is a regime without friction [3], but later it has been shown that eventually friction becomes important [4]. In the present model the substrate is also flexible: both subsystems are crystalline and have phonon excitations. These become coupled which leads to a transfer of energy between the systems, and between the centre of mass motion of one chain and the bath of phonons.

The paper first discusses the coupling of the phonons in the chains of the model in section 2. In section 3 the embedding into a higher-dimensional space is explained, which leads to the possibility of the sliding modes. In section 4 the non-linear effects leading to the onset of friction for sliding modes is discussed.

2. Phonons in the model

As a model for an incommensurate composite we consider a DCM. It consists of two parallel chains of particles. The positions of the particles in the first chain are denoted by x_n, and in the second chain by y_m. The Hamiltonian is given by

$$H = \sum_n \frac{p_{1n}^2}{2m_1} + \sum_m \frac{p_{2m}^2}{2m_2} + \sum_n V_1(x_n - x_{n-1}) + \sum_m V_2(y_m - y_{m-1}) + \lambda \sum_{nm} W(x_n - y_m). \tag{2.1}$$

Because the system is Hamiltonian, the energy is conserved. Furthermore, the Hamiltonian is translation invariant, and hence there is conservation of total momentum, in contrast to the Frenkel–Kontorova-type models.

For $\lambda = 0$ the phonons in the two chains are given by

$$x_n = x_0 + na + u_n = x_0 + na + \sum_k Q_k^{(1)} \exp(\mathrm{i}kna)$$

$$y_m = y_0 + mb + v_m = y_0 + mb + \sum_k Q_k^{(2)} \exp(\mathrm{i}kmb),$$

where the normal coordinates $Q_k^{(j)}$ satisfy

$$\ddot{Q}_k^{(1)} = -\omega_{1k}^2 Q_k^{(1)}$$

$$\ddot{Q}_k^{(2)} = -\omega_{2k}^2 Q_k^{(2)},$$

with $\omega_{1k}^2 = 4\alpha \sin(ka/2)^2$ and $\omega_{2k}^2 = 4\beta \sin(kb/2)^2$ if α and β are the second derivatives of V_j in the equilibrium points divided by m_j. The solutions are given by

$$Q_k^{(j)}(t) = \begin{cases} Q_k^{(j)}(0) \exp(-\mathrm{i}\omega_{jk} t), & \text{if } \omega_k^{(j)} \neq 0 \\ Q_k^{(j)}(0) + vt & \text{else.} \end{cases}$$

If $\lambda \neq 0$ the modes in both chains are coupled. The equations of motion become

$$m_1 \ddot{u}_n = -\alpha(2u_n - u_{n-1} - u_{n+1}) - \lambda \sum_m W'(x_n - y_m)$$

$$m_2 \ddot{v}_m = -\beta(2v_m - v_{m-1} - v_{m+1}) + \lambda \sum_n W'(x_n - y_m).$$

The equations of motion for the normal coordinates, i.e. the Fourier components in the approximation of small u_n and v_m, become

$$\ddot{Q}_k^{(1)} = -\omega_{1k}^2 Q_k^{(1)} - \lambda \sum_{nm} W'(x_n - y_m) \exp(-\mathrm{i}kna)/m_1$$
$$\ddot{Q}_k^{(2)} = -\omega_{2k}^2 Q_k^{(2)} + \lambda \sum_{nm} W'(x_n - y_m) \exp(-\mathrm{i}kmb)/m_2.$$

For small displacements the interaction term may be expanded in powers of $(u_n - v_m)$. Because the function $f(x) = \sum W'(x - mb)$ is periodic in x with period b it has only Fourier components belonging to the reciprocal lattice of the second chain. For similar reasons the first terms in the expansion

$$\sum_{nm} W'(x_n - y_m) \exp(-\mathrm{i}kna) = \sum_{nm} \sum_{j=0}^{\infty} W^{(j+1)}(na - mb)(u_n - v_m)^j/j! \exp(-\mathrm{i}kna)$$

satisfy the relations

$$\sum_{nm} W'(na - mb) \exp(-\mathrm{i}kna) = \sum_{K_2} F_{K_2} \Delta(k - K_2) \tag{2.2}$$
$$\sum_{nm} W''(na - mb) u_n \exp(-\mathrm{i}kna) = \sum_{K_2} \mathrm{i}K_2 F_{K_2} Q_{k-K_2}^{(1)} \tag{2.3}$$
$$\sum_{nm} W''(na - mb) v_m \exp(-\mathrm{i}kna) = \sum_{K_2} \mathrm{i}K_2 G_{K2} Q_{k-K_2}^{(2)} \tag{2.4}$$

and the corresponding terms in the equations of motion for $Q_k^{(2)}$ satisfy similar equations. The terms corresponding to equation (2.2) correspond to a shift of the origin of the vibrations. The particles in chain 1 oscillate around new equilibrium positions corresponding to a modulation of chain 1 with the period of chain 2, and vice versa. The terms corresponding to equation (2.3) give a coupling of the phonons in chain 1 of wavevector k with other modes in the same chain, with wavevector $k + K_2$, where K_2 belongs to the reciprocal lattice of chain 2. In agreement with the mechanism described by Axe [5] for incommensurate phases this coupling between modes $\pm 2\pi/b$ gives rise to the phason branches. The third case, that of the terms corresponding to equation (2.4), gives coupling between the modes in different chains. This coupling is strong if the coupled modes have the same frequency and the same wavevector, modulo the Fourier module spanned by $2\pi/a$ and $2\pi/b$. In an extended zone scheme these cases correspond to crossings of the dispersion curves of both lattices (figure 1, which for reasons of clarity does not correspond to the calculated modes used for figure 2).

For $\lambda = 0$ the phonons are concentrated on either of the two chains. By the coupling, phonons may have lattice displacements in both chains. A measure for the participation of one of the chains is the participation function P_j $(j = 1, 2)$. In calculations the chains are considered in a commensurate approximation. This means that there are integers N and L such that $Na = Lb$. The number of particles in the unit cell is $L + N$. We consider eigenvectors of the mode with wavevector k and label ν: $\epsilon(k\nu|j)$ with normalization

$$\sum_{j=1}^{L+N} |\epsilon(k\nu|j)|^2 = 1. \tag{2.5}$$

Then the participation functions are defined as

$$P_1 = \sum_{j=1}^{N} |\epsilon(k\nu|j)|^2 = 1 - P_2. \tag{2.6}$$

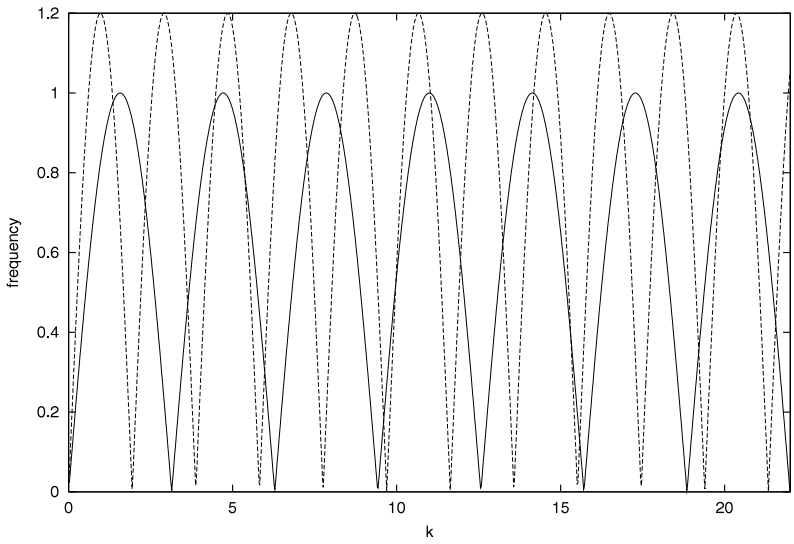

Figure 1. Schematic view of the intersection of the dispersion curves of the non-interacting chains.

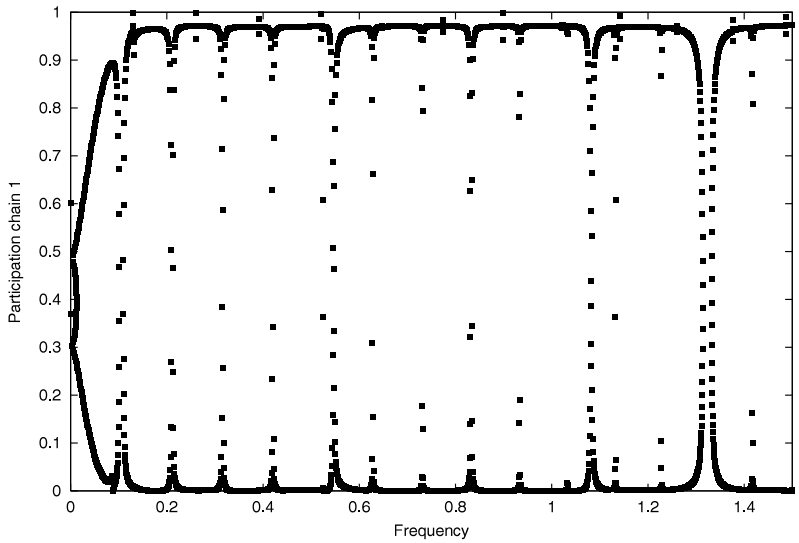

Figure 2. Participation of chain one in the vibration modes of the coupled system.

The value of P_i is between 0 and 1. A value of 0 or 1 means complete concentration on one of the chains. In figure 2 the participation function P_1 is plotted for modes in a commensurate approximant of the DCM. For most of the modes the participation is either 0 or 1. These modes belong to one of the chains. The modes with participation of intermediate value occur for modes near the gaps in the spectrum. They correspond to the values of ω and k where the dispersion curves of the two uncoupled chains cross.

In addition to these coupled modes near crossings there are two branches of coupled modes at low frequency. They correspond to the acoustic modes and the phason modes. The

calculation shows that there are such collective modes in which both chains participate. For a purely acoustic mode at zero frequency all the components $\epsilon(0\nu|j)$ are equal for all j. This implies that $\epsilon(0\nu|j) = (L+N)^{-1/2}$, and consequently

$$P_1 = \frac{N}{L+N}, \qquad P_2 = \frac{L}{L+N}. \tag{2.7}$$

If the interaction λ is small enough, the modulation function is smooth and there is a second zero-frequency mode, the sliding mode. The phonon displacements are proportional to the derivative of the modulation function. For a sinusoidal modulation the result is

$$\epsilon(0\nu|j) = \sqrt{\frac{2L}{N(N+L)}} \qquad j = 1, N \tag{2.8}$$

$$\epsilon(0\nu|j) = -\sqrt{\frac{2N}{L(L+N)}} \qquad j = N+1, N+L. \tag{2.9}$$

The corresponding participation values are $P_1 = L/(L+N)$, $P_2 = N/(L+N)$. In figure 2 the curves do not tend to these values. The reason is that for degenerate modes the participation function is undetermined, because it depends on the ratio of the two degenerate components. Every linear combination of eigenvectors at frequency zero is an eigenvector, and the participation function may reach any value. The numerical procedure gives specific eigenvectors, but they are not unique.

The fact that the participation depends on the frequency is, of course, well known. For the diatomic linear chain with masses m_1 and m_2, P_1 (for particle A) depends on k according to

$$x = \alpha(1+\exp(2\mathrm{i}ka)), \qquad y = 2\alpha - m_2\omega_k^2, \qquad P_1 = |x|^2/(|x|^2+|y|^2).$$

$$\omega_k^2 = \frac{\alpha}{\mu}\left(1 \pm \sqrt{1 - \frac{4\mu \sin{(ka)}^2}{m_1+m_2}}\right), \qquad \mu^{-1} = m_1^{-1} + m_2^{-1}$$

and runs from 0.5 at the zone centre to 0 or 1 at the zone boundary (see figure 3). The difference is that here the modes near the gap are concentrated on one subsystem, and not mixed, whereas in the DCM precisely the modes near gaps are mixed.

3. Embedding

As in every quasiperiodic system the incommensurate double chain may be embedded as a periodic structure in a higher-dimensional space. The equilibrium positions of the particles are given by

$$x_n = x_0 + na + f(x_0+na), \qquad y_m = y_0 + mb + g(y_0+mb), \tag{3.1}$$

where the modulation functions f and g are periodic according to the previous section:

$$f(x+b) = f(x), \qquad g(y+a) = g(y). \tag{3.2}$$

For sufficiently small interaction parameter λ the modulation functions are smooth. For larger values of λ the modulation functions become discontinuous. This is called the discommensuration transition or the transition by breaking of analyticity [3, 6].

The one-dimensional aperiodic structure may be embedded in two dimensions according to

$$(na + f(na-Z), Z), \qquad (Z + mb + g(Z+mb), Z), \tag{3.3}$$

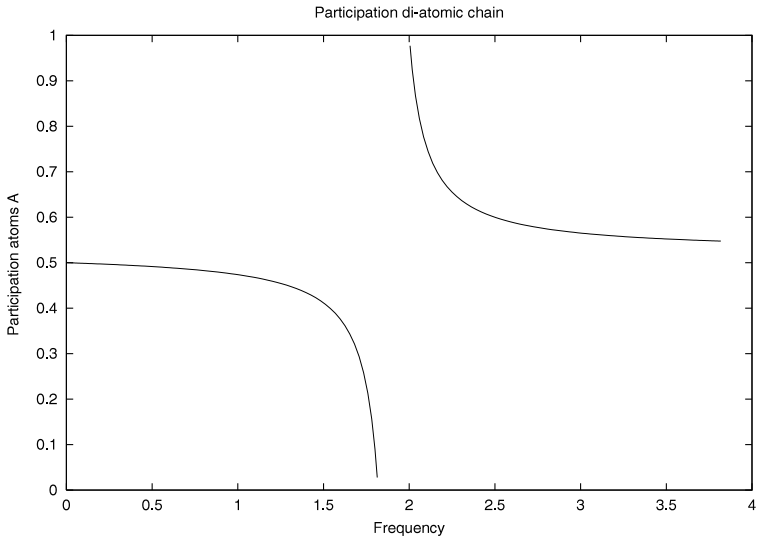

Figure 3. Participation of subsystem A in the diatomic harmonic chain.

where Z is the second coordinate. The array of lines (Z is a real parameter) is invariant under translations (a, a) and $(0, b)$:

$$(na + a + f(na - Z), Z + a) = (n'a + f(n'a - Z'), Z')$$
$$(Z + a + mb + g(Z + mb), Z + a) = (Z' + mb + g(Z' + mb), Z')$$
$$(na + f(na - Z), Z + b) = (na + f(na - Z'), Z')$$
$$(Z + mb + g(Z + mb), Z + b) = (Z' + m'b + g(Z' + m'b), Z').$$

The internal variable Z gives, at the same time, the position of the centre of mass of chain 2 with respect to chain 1. Then a sliding mode corresponds to the shift $Z \to Z+\epsilon$ for some small displacement ϵ. It has been found that the sliding mode has zero frequency if the modulation functions are smooth, but that there is a phason gap in the discontinuous regime [3, 6].

An alternative embedding (figure 4) keeps the total centre of mass unchanged under shifts in Z. Choose parameters t_1 and t_2 with $t_1 + t_2 = 1$ and $t_1/t_2 = am_2/bm_1$ and embed the system according to

$$(-t_1(na + f(na - Z)) + t_2 Z, Z), \qquad (-t_1(Z + mb + g(Z + mb)) + t_2 Z, Z). \tag{3.4}$$

A shift in Z does not change the centre of gravity of the whole system, and the embedding is invariant under a lattice in two dimensions spanned by $(t_1 b, b)$ and $(-t_2 a, a)$. Changing Z causes a relative displacement of the two chains with respect to each other. In the following section we shall study the dynamics connected with this shift.

4. Non-linear dynamics and friction

The vibrations around the equilibrium positions considered in the preceding sections, described in terms of phonons, are harmonic. The harmonic approximation is valid only for small displacements. For larger displacements the equations become non-linear. They are

$$m_1 \ddot{x}_n = -V_1'(x_n - x_{n-1}) - V_1'(x_n - x_{n+1}) - \lambda \sum_m W'(x_n - y_m) \tag{4.1}$$

$$m_2 \ddot{y}_m = -V_2'(y_m - y_{m-1}) - V_2'(y_m - y_{m+1}) + \lambda \sum_n W'(x_n - y_m). \tag{4.2}$$

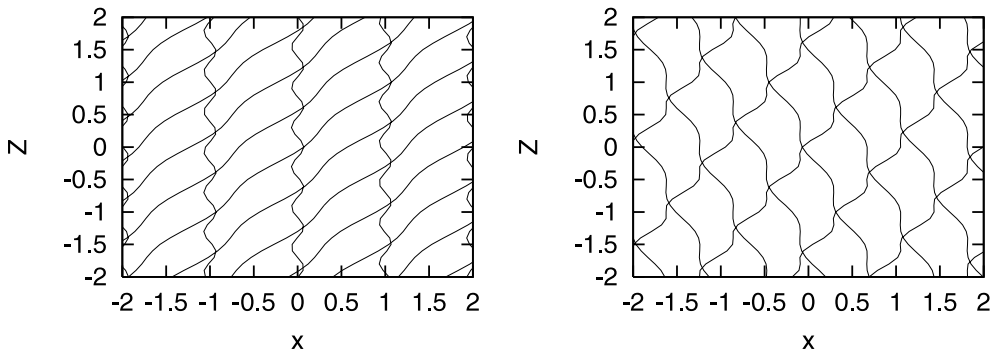

Figure 4. Alternative embeddings of the double chain. For the second, a shift in internal space keeps the centre of mass fixed.

We suppose that the displacements u_n and v_m remain small in the moving frame:

$$V_1'(x_n - x_{n-1}) + V_1'(x_n - x_{n+1}) = \alpha(2u_n - u_{n-1} - u_{n+1})$$

and a similar expression for V_2'. The phonons then are non-linearly coupled by the W terms.

The solutions $x_m(t)$ and $y_m(t)$ determine also the motion of the centre of mass and the motion of the internal coordinate Z. The latter is determined by

$$\begin{aligned} t_2 Z &= \frac{1}{L}\sum_m y_m \rightarrow \\ \ddot{Z} &= \frac{1}{Lt_2}\sum_m \ddot{y}_m = -\frac{\beta}{Lt_2}\sum_m (2y_m - y_{m-1} - y_{m+1}) + \frac{\lambda}{Lt_2}\sum_{nm} W'(x_n - y_m). \end{aligned} \tag{4.3}$$

The relative motion of the two chains may be described as a motion in internal space.

The change in the internal coordinate Z is obtained from the two displacive modes in the two chains:

$$Q_0^{(1)} = \frac{1}{N}\sum_n u_n, \qquad Q_0^{(2)} = vt + \frac{1}{L}\sum_m v_m. \tag{4.4}$$

The equations of motion for these variables are

$$\begin{aligned} \ddot{Q}_0^{(1)} &= -\frac{\lambda}{m_1}\sum_{nm} W'(x_n - y_m) \\ \ddot{Q}_0^{(2)} &= \frac{\lambda}{m_2}\sum_{nm} W'(x_n - y_m). \end{aligned} \tag{4.5}$$

The right-hand sides can be developed in powers of the normal coordinates $Q_k^{(j)}$ according to

$$\sum_{nm} W'(x_n - y_m) = \sum_{nm}\sum_j \frac{1}{j!} W^{(j+1)}(na - vt - mb)(u_n - v_m)^j. \tag{4.6}$$

The first-order terms are given by

$$\begin{aligned} \sum_{nm} W''(na - vt - mb)u_n &= \sum_{K_2 s} \hat{f}_{K_2 s} Q_{K_2}^{(1)} \exp(-\mathrm{i}s\Omega_1 t) \\ \sum_{nm} W''(na - vt - mb)v_m &= \sum_{K_1 s} \hat{g}_{K_1 s} Q_{K_1}^{(2)} \exp(-\mathrm{i}s\Omega_2 t) \end{aligned}$$

where K_1 is a multiple of $2\pi/a$, K_2 is a multiple of $2\pi/b$, $\Omega_1 = 2\pi v/b$ and $\Omega_2 = 2\pi v/a$. The latter frequencies correspond to the frequencies with which the particles of one chain move

over the particles of the other chain. Furthermore, $\hat{f}_{ks}$ and $\hat{g}_{ks}$ are the Fourier transforms of $\sum_m W'(na - vt - mb)$ and $\sum_n W'(na - vt - mb)$, respectively.

By the expressions in terms of the normal coordinates the modes with wavevector k in chain one are coupled to modes at $k + K_2$ in the same chain, and to modes at $k + K_1$ in the other chain, and vice versa. The centre of mass motions are in first approximation coupled to modes with wavevectors in one of the two reciprocal lattices. Then the equations of motion become

$$\ddot{Q}_0^{(1)} = -\frac{\lambda}{m_1}\left[\sum_{K_2} \hat{f}_{K_2 1} Q_{K_2}^{(1)} \exp(-\mathrm{i}\Omega_1 t) + \sum_{K1} \hat{g}_{K_1 1} Q_{K_1}^{(2)} \exp(-\mathrm{i}\Omega_2 t)\right]$$

$$\ddot{Q}_0^{(2)} = \frac{\lambda}{m_2}\left[\sum_{K_2} \hat{f}_{K_2 1} Q_{K_2}^{(1)} \exp(-\mathrm{i}\Omega_1 t) + \sum_{K1} \hat{g}_{K_1 1} Q_{K_1}^{(2)} \exp(-\mathrm{i}\Omega_2 t)\right]$$

$$\ddot{Q}_{K_2}^{1} = -\omega_{1K_2} Q_{K_2}^{(1)}$$

$$\ddot{Q}_{K_1}^{2} = -\omega_{2K_1} Q_{K_1}^{(2)}.$$

The solutions give for the change in the internal coordinate

$$Z_1 - Z_2 \approx vt + A\exp(-\mathrm{i}\Omega_1 t \pm \mathrm{i}\omega_{1K_2} t) + B\exp(-\mathrm{i}\Omega_2 t \pm \mathrm{i}\omega_{2K_1} t). \tag{4.7}$$

The result is a quasiperiodic oscillation of the internal coordinate around a mean value v. Through coupling to other modes energy flows from this centre of mass motion to the phonon bath. The flow is most important in the regions where one of the frequencies $s\Omega_i$ becomes equal to a frequency ω_{1K_2} or ω_{2K_1}. The effect is even more pronounced at frequencies where the participation of the chains is comparable in size (i.e. approximately 0.5).

The analysis given above may be illustrated by numerical calculations. This allows us to explore the region that is not accessible to analytical treatment. We consider the DCM with truncated Lennard-Jones potentials:

$$V_i(x) = ((a_i/x)^{12} - 2(a_i/x)^6)\exp(-rx^2).$$

Chain lengths up to $N = 89$ and $L = 144$ were considered, with periodic boundary conditions. The equations of motion were integrated with a four-step Runge–Kutta procedure. For various values of the lengths L and N the equations were integrated with the equilibrium positions as positional initial conditions, zero velocity for the particles of chain one, and a uniform initial velocity of the particles of chain two. The monitored properties were the momenta of the two chains and their kinetic energies as a function of time.

In the first simulations the inter-chain coupling was taken to be so small that the modulation functions were smooth. In figure 5 the momentum of chain two is plotted as a function of time for a number of initial velocities. For momentum smaller than a critical value $v = 1.1$ it remains practically constant for a very long time. For $v \approx 0.65$ the coupling becomes stronger, the energy is lost faster and there are stronger oscillations due to the resonance of Ω_1 and ω_{1K_2}. The resonance disappears for higher values of v. Above the critical value the energy loss is much stronger. There is no longer a sliding mode. If the relative motion vanishes the momentum of the second chain goes to $L/(L + N)$ of its original value because of conservation of total momentum. Figure 6 shows the kinetic energy in both chains as a function of time, when there is a strong dissipation ($v = 1.5$). Chain 2 quickly loses its kinetic energy to chain 1, until the point that the energy is evenly distributed over the modes of both chains. The cross-over from almost dissipationless to strong dissipative behaviour is very similar to that in the Frenkel–Kontorova model for weak coupling [4]. The calculations show that, for low velocities, the energy loss, and therefore also the damping of harmonic modes, is very small. In experiments the phason and sliding modes have been found usually as strongly

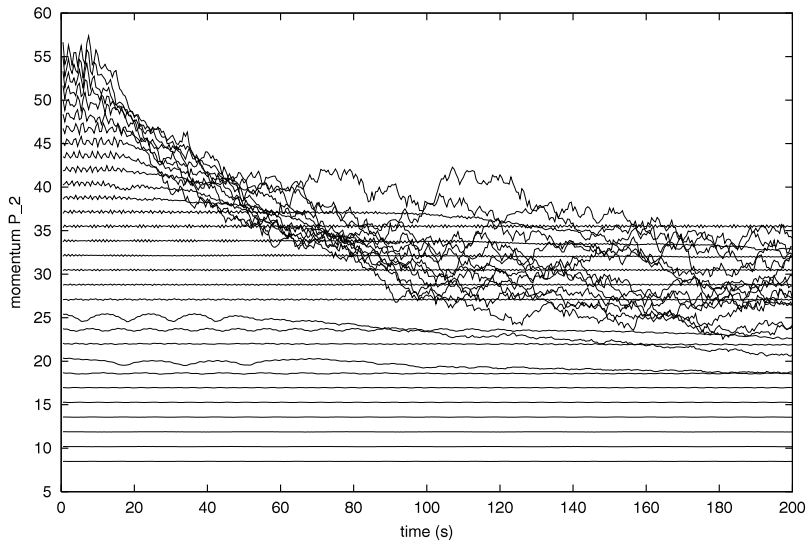

Figure 5. Momentum of the second chain as a function of time for initial velocities from 0.5 (lowest curve) up to 2.0 (highest curve) with intervals equal to 0.05.

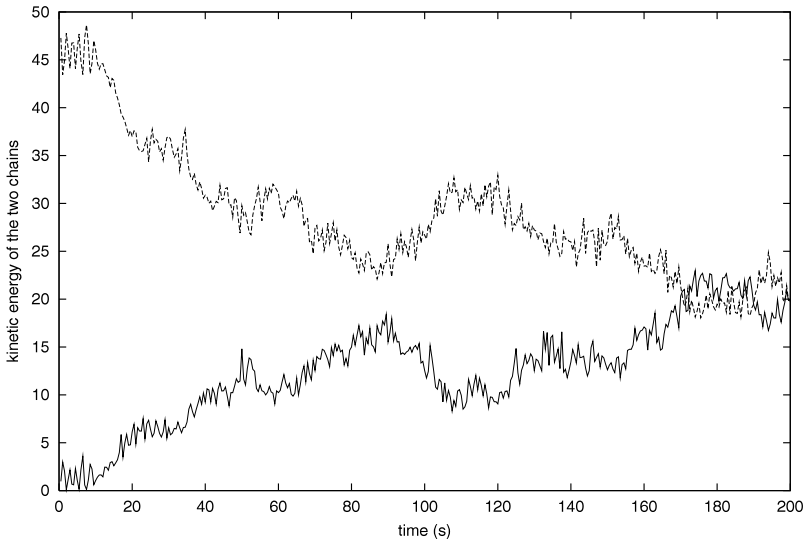

Figure 6. The kinetic energies of the chains as a function of time for the relative initial speed $v = 1.5$. The systems tends to the equilibrated state.

damped. This would then be not an intrinsic property of the dynamics of incommensurate phases, but probably due to other effects, such as the coupling to defects and pinning. In [7] the dynamics of incommensurate phases has been studied with a phenomenological approach to the damping.

When the coupling between the chains becomes stronger the modulation functions are no longer continuous, and the analysis in terms of normal coordinates of the two chains is

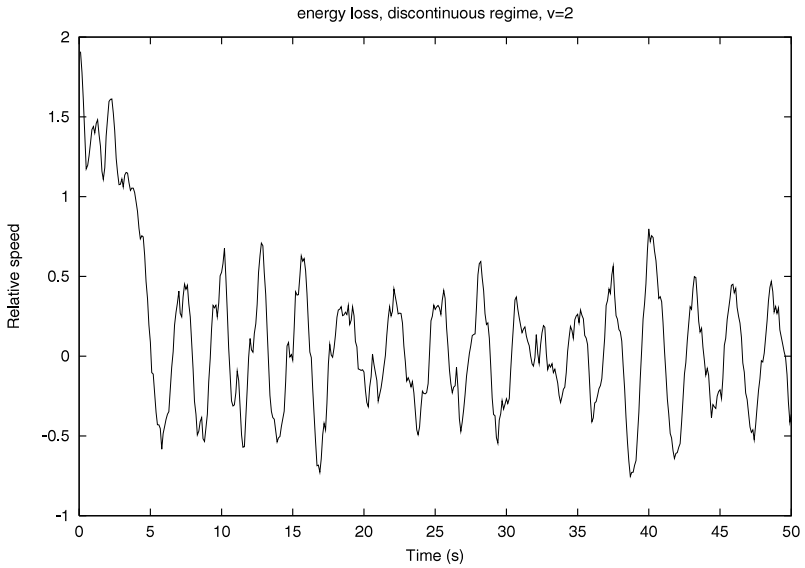

Figure 7. The relative speed of the two chains as a function of time, for an inter-chain coupling large enough to give a discontinuous modulation function. The initial speed is small and the motion is pinned.

no longer valid. The two chains are still mutually incommensurate, which means that the ground state remains infinitely degenerate. However, in this case there are barriers between the various ground states involving finite jumps of the particles. Therefore, the displacements are no longer harmonic. A numerical integration of the equations of motion gives another cross-over behaviour. For low relative momenta the kinetic energy is not sufficient to cause the particles to move over the barriers. Then the kinetic energy is exchanged between the two subsystems and the centre of mass oscillates (figure 7). For higher momenta the two chains may slide over each other, and the kinetic energy is quickly transferred to the phonon degrees of freedom (figure 8). The transfer of energy between the two chains for the pinned and unpinned cases is illustrated in figure 9. The initial velocity $v = 2.0$ is below the critical value, whereas $v = 3.3$ is above it.

5. Concluding remarks

The coupling in the DCM, a model for incommensurate composites or the dynamics of contacting surfaces, leads to a mutual modulation of the two chains and to a renormalization of the phonons. For small inter-chain interactions the modulation is smooth, but for stronger interactions the modulation functions become discontinuous. In the smooth regime there are vibrational excitations concentrated on one of the two chains, but near gaps and for low frequency there are modes with a comparable contribution of both chains. For low frequencies two branches appear in which both chains participate, one with the character of acoustic waves, the other with a phason-like character. The latter branch has no gap when the modulation is smooth, and the excitations have a displacement component in the internal space. For larger inter-chain interaction the modulation functions are discontinuous and a phason gap opens up. The static friction coefficient is zero when the phason gap is closed, and becomes non-zero when the phason gap opens.

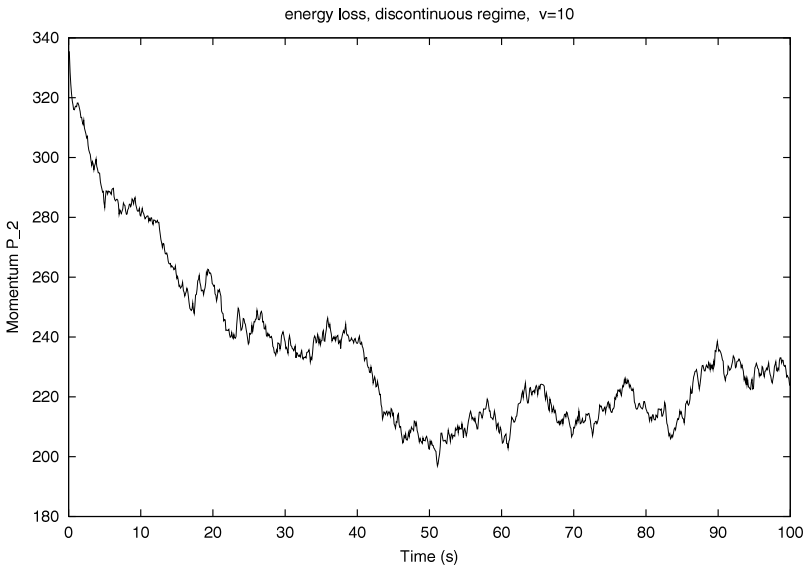

Figure 8. The momentum of the second chain as a function of time, for an inter-chain coupling large enough to give a discontinuous modulation function. The initial speed is high and the motion unpinned. The final momentum corresponds to zero velocity in internal space (no relative speed).

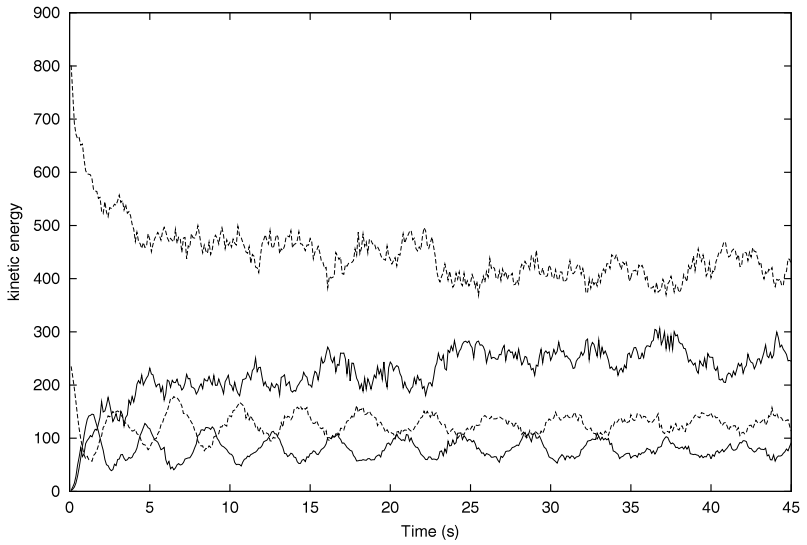

Figure 9. The kinetic energies for both chains in the discontinuous regime for speeds $v = 2.0$ and 3.3. Full curves: chain 1, broken curves: chain 2.

In the continuous regime the amplitudes of the phason may become large. There is a cross-over from a regime at low sliding speeds where the sliding mode interacts only weakly with the phonons to a regime where the kinetic energy of the sliding mode is quickly lost to phonon excitations. In the low speed limit the dissipation, i.e. the energy transfer to phonons, is low, but non-zero. In terms of friction this is the pseudo-superlubric state with a very low dynamic friction coefficient. Resonances between the beat frequencies of the sliding mode

and the phonon frequencies give rise to a higher dissipation for special values of the sliding velocity. The transition from an (almost) dissipationless motion to strong dissipation has been found also in the dynamics of incommensurate modulated phases. In the discrete frustrated ϕ^4 (DIFFOUR) model the non-linear solitary wave becomes unstable for increasing velocities [8]. Observations of low friction in such situations have been reported by Hirano *et al* [9] in STM experiments.

For a strong inter-chain interaction, when the modulation is discontinuous, there is a transition from a pinned state to an unpinned state for increasing sliding velocities. In the high speed regime the dissipation increases strongly.

The present very simple model for the dynamics of incommensurate composites in physical and internal space neglects a number of important features, such as the influence of temperature and defects. Moreover, increasing the dimension may introduce new phenomena.

References

[1] Radulescu O and Janssen T 1997 *J. Phys. A: Math. Gen.* **30** 4199
[2] Radulescu O and Janssen T 1998 *Aperiodic '97* ed M de Boissieu, J-L Verger-Gaugry and R Currat (Singapore: World Scientific)
[3] Shinjo K and Hirano M 1993 *Surf. Sci.* **283** 473
[4] Consoli L, Fasolino A, Knops H and Janssen T 2001 *Ferroelectrics* **250** 111
[5] Axe J D 1980 *Phys. Rev.* B **21** 4181
[6] Brussaard L A, Fasolino A and Janssen T 2001 *Phys. Rev.* B **63** 214302
[7] Currat R, Kats E and Luk'yanchuk I 2002 *Eur. J. Phys.* B **26** 339
[8] Janssen T, Radulescu O and Rubtsov A N 2002 *Eur. Phys. J.* B **29** 85
[9] Hirano M, Shinjo K and Kaneko R 1997 *Phys. Rev. Lett.* **78** 1448